ECON4930 Seminar exercise 1, Spring 2011

1. Consider the following social planning problem for utilisation of water over time within a hydro generating system given the past investment in generation capacity, transmission network, etc.

$$\max \sum_{t=1}^{T} \int_{z=0}^{e_t^H} p_t(z)\,dz$$

subject to

$$R_t \le R_{t-1} + w_t - e_t^H$$

$$R_t \le \overline{R}$$

$$R_t, e_t^H \ge 0$$

$$T, w_t, R_o, \overline{R} \text{ given, } R_T \text{ free, } t = 1,..,T$$

where

e_t^H = electricity production during period t (kWh)

$p_t(e_t^H)$ = demand function

R_t = reservoir level at end of period t (kWh)

w_t = inflow during period t (kWh)

$\overline{R}$ = reservoir constraint (kWh)

a) Explain the relations in the planning problem, starting with the objective function, and comment upon underlying assumptions.

b) Derive the first-order conditions for the problem. Make economic interpretations introducing reasonable assumptions.

c) Discuss qualitative aspects of possible optimal solutions regarding electricity prices, focussing on possibilities for price changes. In your discussion try to make use of the principle of backward induction, i.e. starting with period T (hint: find the optimal solution for period T, then move to period T-1, etc. Try to illustrate your discussion using bathtub diagrams, and make sure that the diagrams conform with the first-order conditions.

d) Introduce a constraint on the generation of electricity $e_t^H \le \overline{e}^H$. Explain the interpreation of the constraint, and study the consequence for the possibility of price changes of the new optimal solution (hint: introduce the constraint in the problem in question a), and expand the Lagrangian function correspondingly). Try to illustrate using a bathtub diagram.